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COMMON STOCHASTIC TRENDS AND EFFICIENCY IN THE FOREIGN EXCHANGE MARKET[*]

Abstract. *Within the traditional financial literature about efficient markets there are two interpretations on empirical results, some works find evidence of the predictive superiority of the martingale (martingale hypothesis); while others provide evidence supporting a hypothesis of cointegration among exchange rates. In this context, we analyze triangular arbitrage cointegration relationship, among Dollar-US, Euro and others 20 currencies, to test efficiency market of exchange rates. For this, we consider implicit transaction costs in daily Ask-Bid cointegration relationships with heteroskedasticity covariance matrix, and using alternative methods: cointegration relations with and without heteroskedasticity, and dynamic factorial analysis. We do not find empirical evidence of the lack of arbitrage opportunities, so the results support efficiency of the exchange markets.*

Key words: *efficient market, cointegration, opportunities arbitrage, transaction cost, BEKK.*

JEL Classification: C32, F31, G14.

1. Introduction

As world trade needs to know what the equilibrium exchange rates in the long term, with the disadvantage that fluctuations in the short-term market exchange rate could not be full explained by macro variables, in the financial literature arise works on the foreign exchange market efficiency. This question has been studied from three perspectives.

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First, to study when exchange rates have been in a state of equilibrium respect interest rates and price levels. These works estimate a time-varying behavioural equilibrium exchange rate testing differences theories as Purchasing Power Parity (PPP), Covered Interest Parity (CIP), Expectations Theory (ET), Uncovered Interest Parity (UIP) and Fisher Effect (FE) hypothesis (among others: Caporale *et al.*, 2001).

Second, to research if the forward exchange rate is an unbiased predictor of future spot exchange rates, i.e., forward rate unbiasedness hypothesis (see: Baillie and Bollerslev, 1989).

Third, our study matter, to test the spot exchange market efficiency (see: Diebold *et al.*, 1994; Crowder, 1994). For this goal, the empirical works use cointegration techniques to analysis the triangular arbitrage relation. The results have two possible interpretations:

- If there is not cointegration relationship, the foreign exchange market is efficient, that is, changes in exchange rates are unpredictable. In other words, if one could forecast a exchange rate on the basis of other one in the cointegrated system, then hypothesis martingale would be violated (Diebold *et al.*, 1994).
- Efficiency and cointegration are not opposite, as argue Crowder (1994), among others. Thus, a market is efficient when there are no risk-free returns above opportunity cost.

The initial empirical studies on triangular arbitrage used monthly or weekly data. Dutt and Ghosh (1999) for weekly data from the New York Money Market and International Financial Statistics Monthly, from January 1973 to April-1994, rejected that markets are efficient. Since the market operations are becoming increasingly automated, the most recent studies analyze daily or high frequency data.

In this context, Martens and Kofman (1998), contrasting data from the Reuters platform FXFX high frequency (1-3 minutes) with the currency forward prices on the Chicago Mercantile Exchange, found apparent arbitrage opportunities against the forward-spot price Reuters, concluding that the use of different data sources could lead to conflicting results. Kollias and Metaxas (2001) used data tick to tick, concluding that arbitrage opportunities have very short duration and, cannot be regarded as market inefficiencies, but to an effect of slippages of currency market quotes. Marshall and Treepongkaruna (2008), with high frequency data (less than 5 minutes) from the EBS platform, found evidence of small triangular arbitrage opportunities, which increasing the observation time of the data disappear. Fong *et al.* (2008) found evidence of arbitrage opportunities in high frequency data for spot and forward exchange rate of US-Dollar and Hong Kong- Dollar, but they noted that the theoretical benefits occur in low market liquidity and likely, transaction costs would offset it mostly. Fenn *et al.* (2009) found arbitrage opportunities in the spot market exchange rate by triangular arbitrage operations in high-frequency observations obtained by Reuters to refresh 5 minutes, concluding that the market is not efficient, but these opportunities are more than 94% of cases of a magnitude less than 1 b.p. (basic point) and with a duration of less than 1 second, further such opportunities are manifested in extreme hours from GMT (Greenwich Mean Time). Moore and Payne (2011) show that the volume of the operation and size of the operator are two significant variables of arbitrage opportunities, and in the case of cross-exchange market, they find that the cross-traders are the group operators with better information, so one might conclude that the arbitrage opportunities, if any, are limited by the size and time. Kozhan and Tham (2012), found evidence of triangular arbitrage opportunities are not exploited immediately, so they conclude those either are not optimal either not financially compensate the operators. Chung and Hrazdil (2012) show that price adjustments to new information do not occur instantly.

Additionally, for high frequency data arise the synchronization problem among the prices of different financial assets. Akram *et al.* (2008) found evidence of arbitrage opportunities in high frequency data, but to solve the problem of synchronization, chose to work with the last price offered in each case (stale quotes), without considering further that to close an operation in a market, operators require a minimum time. Gagnon and Karolyi (2010) show that using illiquidity and stale quotes could lead to erroneous conclusions because of the intrinsic problems of the original data, resulting in spurious arbitrage opportunities.

Besides above problems, as the literature has progressed, it has moved from considering only price bid, to analyze the prices adjusted to the reality of trading in the foreign exchange market. As we know the market-makers must give orders to ask and bid prices simultaneously (two-way or round-trip quotes[1]). However, the methodological approach of many papers does not analyze jointly the behavior of both prices, operating with the spread or both prices separately. For example Fenn *et al.* (2009) estimated separately ask and bid price and then, compare theses prices to contrast arbitrage opportunities, so they obviate the multivariate distribution of the 6 variables (2 prices, ask-bid for 3 currencies). Others, as Choi (2011), uses a theoretical average spread, resulting from the observed spread and unobserved weighting, also left aside the long-term relationship or equilibrium among the exchange rate variables (not stationary), since operates with relative variation rates (stationary).

In summary, we observe recurrent problems in the studies about arbitrage opportunities on high frequency data: the minimum margin, the short duration, in extreme hours from GMT, the low liquidity for potential arbitrage opportunities, the stale quotes and the failure to consider the bid-ask spread.

[1] Round-trip and one-way arbitrage conditions differ in that violations of the latter do not necessarily prove the existence of riskless profits, so one-way requires an excess supply or demand of fund, while round-trip not. As consequence, one-way arbitrage opportunities only indicate quote differentials that are due to different practices and market segmentation

Therefore, the aim of this paper is to study if cointegration and efficiency are possible simultaneously in foreign exchange market, or if there are arbitrage opportunities. For that, we identify the structure of Vector Error Correction Model (VECM) of triangular arbitrage among 20 daily spot prices against Dollar-USA (USD) and Euro (EUR) from a single platform (Reuters), thus avoiding the problems synchronization, and using Ask and Bid exchange rate, this allows to reply more exactly hedged portfolio and to include the implicit transaction costs.

The remainder of this paper is organised as follows. Section 2 reviews the financial relations of the model. Section 3 shows the econometric methodology used. In Section 4 we present the data. The results are in Section 5. Finally, the conclusions are in Section 6.

2. The Triangular Arbitrage

The cross exchange or triangular arbitrage is defined as:

$$\left(\frac{C_i}{C_j}\right)_t \equiv \left(\frac{C_i}{C_h}\right)_t \times \left(\frac{C_h}{C_j}\right)_t \tag{1}$$

Where (C_i, C_j, C_h) are three currencies for a day *t* and its three possible spot exchange rates are expressed as a ratio.

So, if we operate with three currencies and their respective bid and ask prices, there are six triangular relations among them: three relationships of ask type and other three bid type. Among them, two are usually studied in isolation from the rest: Forward Arbitrage (bid = ask x ask) and Reverse Arbitrage (ask = bid x bid).

In a frictionless market, taking logarithms in (1) to obtain a linear triangular arbitrage relationship, results:

$$ln\left[\left(\frac{C_i}{C_j}\right)_t\right] = ln\left[\left(\frac{C_i}{C_h}\right)_t\right] + ln\left[\left(\frac{C_h}{C_j}\right)_t\right] \tag{2}$$

When we consider frictions, then implicit transaction costs suppose that there are two simultaneous exchange rates (Ask and Bid prices) and the relation (2) is:

$$ln\left[\left(\frac{C_i}{C_j}\right)_{t,\psi}\right] = ln\left[\left(\frac{C_i}{C_h}\right)_{t,\psi}\right] + ln\left[\left(\frac{C_h}{C_j}\right)_{t,\psi}\right] \qquad \psi = \{a, b\} \tag{3}$$

Where *a* is Ask price and *b* Bid price.

In order to obtain triangular arbitrage relations considering both market sides (offer and demand), we include the implicit transaction cost rate $(\mu_{i,j})$. So, implicit transaction cost is: $exp\left[\frac{(C_i/C_j)_{t,b}}{(C_i/C_j)_{t,a}}\right]$.

Then, we express the Ask-Bid relation between two currencies as follow:

$$ln\left[\left(\frac{C_i}{C_j}\right)_{t,b}\right] = \mu_{i,j,t} + ln\left[\left(\frac{C_i}{C_j}\right)_{t,a}\right] \tag{4}$$

The expected value of transaction cost rate is $(\mu_{i,j} > 0)$, because dealers buy cheap and sell expensive.

3. Econometric Methodology

As our aim is testing exchange rate market efficiency, we contrast if there are arbitrage opportunities in the long-run triangular arbitrage relation. For this, the general expression for arbitrage strategies (3) becomes the following econometric model:

$$\begin{gathered} ln\left[\left(\frac{C_i}{C_j}\right)_{t,\psi}\right] = \delta^{\psi}_{i,j} + \lambda^{\psi}_{i,h} ln\left[\left(\frac{C_i}{C_h}\right)_{t,\psi}\right] + \lambda^{\psi}_{h,j} ln\left[\left(\frac{C_h}{C_j}\right)_{t,\psi}\right] + u^{\psi}_{i,j,t} \\ u^{\psi}_{i,j,t} \sim iid\left(0, \varpi^2_{i,j}\right) \qquad \forall i \neq j \neq h \qquad \psi = \{a, b\} \end{gathered} \tag{5}$$

Where $\delta^{\psi}_{i,j}$ is the long-run result of triangular arbitrage for currency *i* with ψ price. The parameters $\lambda^{\psi}_{i,h}$ and $\lambda^{\psi}_{h,j}$ are the weights in reply portfolio of currencies *h* and *j*, respectively and, its expected value according to (3) is $\left|\lambda^{\psi}_{*,h}\right| = 1$.

In the same way, Ask-Bid relation (4) becomes:

$$ln\left[\left(\frac{C_i}{C_j}\right)_{t,b}\right] = \mu_{i,j} + \phi_{i,j} ln\left[\left(\frac{C_i}{C_h}\right)_{t,a}\right] + \varepsilon_{i,j,t} \qquad \varepsilon_{i,j,t} \sim iid\left(0, \sigma^2_{i,j}\right) \tag{6}$$

Where expected value according to (4) is $\phi_{i,j} = 1$.

In presence of transaction cost, we have Bid and Ask prices at the same time, and if we trade on both sides of the market, as occur in the trading arbitrage strategies, the no arbitrage condition with transaction costs implies: $\mu_{i,j} > 0$, $\delta^{\psi}_{i,j} \neq 0$ and $\mu_{i,j} - \delta^{\psi}_{i,j} = 0$. That is, if there are not arbitrages opportunities, the transaction costs of both sides would be netting.

In this context, the no arbitrage condition implies the existence of a cointegration relationship among exchange rate of (C_i, C_j, C_h) currencies. To verify this it is sufficient to analyze the relationship among the error terms of (5) and (6):

$$\begin{aligned} \varepsilon_{i,j,t} &= -u^{a}_{i,h,t} + u^{b}_{h,j,t} = u^{b}_{i,h,t} - u^{a}_{h,j,t} \\ \varepsilon_{i,h,t} &= -\left(u^{a}_{i,j,t} + u^{a}_{h,j,t}\right) = u^{b}_{i,j,t} + u^{b}_{h,j,t} \\ \varepsilon_{h,j,t} &= -\left(u^{a}_{i,j,t} + u^{a}_{i,h,t}\right) = u^{b}_{i,j,t} + u^{b}_{i,h,t} \end{aligned} \tag{7}$$

In practice, this result is employed to test market efficiency. For so doing, we express the problem of efficiency in VECM form as:

$$\Delta X_t = \Pi \cdot X_{t-1} + \sum_{d=1}^{k} \delta_d \cdot \Delta X_{t-d} + \varepsilon_t \quad (8)$$

Where X is a vector (with dimension: 6 rows and 1 column) of the logarithms exchange rate Ask and Bid for three currencies (C_i, C_j, C_h). (Δ) is the first difference operator, and (Π) is a matrix whose rank (r) indicates the number of cointegration relationship. The decomposition of the Π matrix is: $\Pi = \alpha \cdot \beta'$, where α is the adjustment coefficients matrix and β is the coefficient matrix of the cointegration relationship.

The interpretations of k and r in (8) are the usual in cointegration context:

- If k=1 and r=0 then, the exchange rate is a pure martingale.
- If k>1 and r=0 then, the exchange rate has a dynamic behaviour.
- If k>1 and r>0 then, there are (r) stationary relations and the process is long memory. For this case, we study if efficiency and cointegration are opposite or not.

The expression (8) could be estimated by applying the methodology of Johansen (1988) and using AIC (Akaike Information Criterion) and SIC (Schwartz Information Criterion) to select the optimal lags (k).

Cheung and Lai (1993) showed that robustness of Johansen´s LR tests (log-likelihood ratio) depends on a suitable selection of lags, and when data is moving-average, the information criterions perform poorly in selecting the optimal lag. Additionally, Johansen's LR test assumes homoskedasticity innovations, but the stylized facts of high frequency financial data in exchange rate (Bos *et al.*, 2000) show non-Gaussian innovations and heteroskedasticity. For these cases, Cavaliere *et al.* (2010), from the previous work of Swensen (2006), propose a pseudo-LR test that, using wild bootstrap technique, allows contrasting cointegration rank for a VAR-MGARCH process.

As mentioned above, we expect to find two cointegration relationships (forward and reverse) among each set of six exchange rates, then we define the next VAR(k)-BEKK GARCH(1,1) to guarantee that variance is always positive definite:

$$\Delta X_t = \Pi \cdot X_{t-1} + \sum_{d=1}^{k} \delta_d \cdot \Delta X_{t-d} + \varepsilon_t =$$
$$= \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \vdots & \vdots \\ \alpha_{6,1} & \alpha_{6,2} \end{bmatrix} \cdot \begin{bmatrix} \beta_{0,1} & \cdots & \beta_{6,1} \\ \beta_{0,2} & \cdots & \beta_{6,2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \vdots \\ X_{6,t-1} \end{bmatrix} + \sum_{d=1}^{k} \delta_d \cdot \Delta X_{t-d} + \varepsilon_t$$
$$\varepsilon_{t,6\times1} = \xi_{t,6\times1} \cdot \Omega_{t,6\times6}^{\frac{1}{2}} \qquad \xi_{t,6\times1} \sim iid(0,1) \tag{9}$$
$$\Omega_{t,6\times6} = \begin{bmatrix} \sigma_{1,1,t} & \cdots & \sigma_{1,6,t} \\ \vdots & \ddots & \vdots \\ \sigma_{6,1,t} & \cdots & \sigma_{6,6,t} \end{bmatrix} = C_{0,6\times6} \cdot C'_{0,6\times6} + A_{1,6\times6} \cdot \varepsilon'_{t-1,6\times1} \cdot A'_{1,6\times6} + B_{1,6\times6} \cdot \Omega_{t-1,6\times6} \cdot B'_{1,6\times6}$$

In (9) for each currency *i* and at instant *t*: ΔX_t is a vector of daily returns or first difference of the exchange rates logarithms (6 rows and 1 column, i.e. the exchange rate ask and bid for USD/EUR, for each currency/USD and for each currency/EUR); $\alpha_{*,r}$ shows the adjustment by *r* cointegration relationship on variations in exchange rate *i*; X_{t-1} is log-exchange rate at instant *t-1*, the participation of this exchange rate in each cointegration relationship (ask or bid) is defined by parameters with known values [-1, 0, 1] depending on rate type and how it was expressed in (3), so VAR-BEKK with constraints is:

$$\beta = \begin{bmatrix} \beta_{0,1} & 1 & -1 & -1 & 0 & 0 & 0 \\ \beta_{0,2} & 0 & 0 & 0 & -1 & 1 & 1 \end{bmatrix} \tag{10}$$

Also, the parameters $\beta_{0,1}$ and $\beta_{0,2}$ are transaction cost rates, then the null hypothesis of the lack of arbitrage opportunities is: $(\beta_{0,1} - \beta_{0,2} = 0)$.

Additionally, MGARCH matrix are:

$$C_0 = \begin{bmatrix} c_{1,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ c_{6,1} & \cdots & c_{6,6} \end{bmatrix} \quad A_1 = \begin{bmatrix} a_{1,1} & \cdots & a_{1,6} \\ \vdots & \ddots & \vdots \\ a_{6,1} & \cdots & a_{6,6} \end{bmatrix} \quad B_1 = \begin{bmatrix} b_{1,1} & \cdots & b_{1,6} \\ \vdots & \ddots & \vdots \\ b_{6,1} & \cdots & b_{6,6} \end{bmatrix} \tag{11}$$

Now, following Cavaliere *et al.* (2010), we define:

$$Z_0 = \Delta X_t \quad Z_r = X_{r,t-1} \quad Z_k = \Delta X_{t-k}$$
$$M_{i,j} = \frac{1}{T} \sum_{t=1}^{T} Z_i \cdot Z'_j \qquad i,j = 0,r,k \tag{12}$$
$$S_{i,j} = M_{i,j} - M_{i,k} \cdot M_{k,k}^{-1} \cdot M_{k,j} \qquad i,j = 0,r$$

Then, LR test for cointegration rank is from log-likehood function:

$$\ell(r) = -\frac{T}{2}\left[ln|S_{0,0}| + \sum_{i=1}^{r} ln(1-\lambda_i)\right] = -\frac{6T}{2} ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T} ln|\Omega_t| + -\frac{1}{2}\sum_{t=1}^{T} \varepsilon'_t \cdot \Omega_t^{-1} \cdot \varepsilon_t$$
$$LR\ test(r) = -2 \cdot [\ell(r) - \ell(6)] = -T \cdot \sum_{i=r+1}^{N} ln(1-\lambda_i) \qquad \forall\, r \leq 5 = 6-1 \tag{13}$$

Where $\ell(r)$ is the maximized log-likelihood under a cointegration rank-*r*, and λ are the eigenvalue solving:

$$\left|\lambda \cdot S_{r,r} - S_{r,0} \cdot S_{0,0}^{-1} \cdot S_{0,r}\right| = 0 \qquad \lambda_1 > \lambda_2 > \cdots > \lambda_r > \cdots > \lambda_6 \tag{14}$$

So, bootstrap algorithm steps are:

1. Fit a cointegrated VAR-BEKK without constraints and estimate the residuals. Next, fit cointegrated VAR-BEKK with constraints on the cointegration rank. And calculate LR-test of cointegration rank. This test is distributed as $\chi^2_{r\times(6-s)}$, where *s* is the number of constraints on matrix Π.
2. Generate *T* bootstrap residuals: $\varepsilon_t^{bt} = \varepsilon_t \cdot \omega_t \quad \omega_t \sim Normal(0,1)$.
3. Construct the bootstrap sample recursively from: $\Delta X_t^{bt} = \Pi \cdot X_{t-1}^{bt} + \sum_{d=1}^{k} \delta_d \cdot \Delta X_{t-d}^{bt} + \varepsilon_t^{bt}$.
4. Using bootstrap sample to obtain bootstrap LR-test from $\ell^{bt}(r)$ and $\ell^{bt}(6)$.
5. Repeat 2-4, *M* times[2].
6. Calculating p-value as: $p_r^{bt} = \frac{1}{M}\sum_{m=1}^{M} 1\left(LR_{r,m}^{bt} > LR_r\right)$. To reject the null hypothesis at a significance level (α), if $p_r^{bt} < \alpha$.

Besides, as an analysis of robustness of the results of bootstrap, for a set of variables (*N*) we estimate the cointegration rank (*r*) from the number of common factors (*m*) as *r* = *N*-*m* (see: Gonzalo and Granger, 1995). In this case, the problem formulation would depend on a number of factors (*F*) and a load matrix (γ):

$$\begin{aligned} X_t &= \gamma \cdot F_t + \omega \cdot \varepsilon_t \\ F_t &= \rho \cdot F_{t-1} + u_t \end{aligned} \qquad (15)$$

Let us note that if $|\rho|$ is less than 1, the factor is *I*(0) or stationary, by the contrary, the factor is *I*(1) on non-stationary when the value is one. Among dynamic common factors methodologies, Peña and Poncela (2006) (PP henceforth) express the factorial model as follows:

$$X_t = \mu + [\gamma_1 \quad \gamma_2] \cdot [F_{1,t} \quad F_{2,t}]' + \varepsilon_t \qquad (16)$$

That is, if there are *N* observed variables with the corresponding mean vector (μ), then the common factors explaining may belong to the subset F_1 of factors *I*(1) or the F_2 subset of common factors *I*(0). Thus, if the first group is made up m_1 factors and the second m_2, then there must be $r=N-(m_1+m_2)$ cointegration relationships.

The advantage of this framework is that, regardless of whether the observed series are stationary or not, can identify stationary and non-stationary common factors. It also allows to test whether the cointegration test correctly discriminate between cointegration relations and stationary factors.

The test is implemented as follows:

$$\begin{aligned} Y_t &= X_t - \mu \\ \widehat{M}(k) &= \left[\sum_{t=k+1}^{T} Y_t \cdot Y_t'\right]^{-1} \cdot \left[\sum_{t=k+1}^{T} Y_t \cdot Y_{t-k}'\right] \cdot \left[\sum_{t=k+1}^{T} Y_{t-k} \cdot Y_{t-k}'\right]^{-1} \cdot \left[\sum_{t=k+1}^{T} Y_{t-k} \cdot Y_t'\right] \end{aligned} \qquad (17)$$

[2] We simulated 5,000 times. The average computation time per currency was 127:55:12 (hours:min:sec), while the total for the 20 currencies was 2,558.46 hours (aprox. 105 days). We use a DELL Precision M 6500 mobile workstation with 32GB (RAM), Intel Core i7 (processor) and two HDDs of 465 GB each.

According to Peña and Poncela (2006) Theorem-3, the matrix $\widehat{M}(k)$ has *N-(m₁+m₂)* eigenvalue which converge in probability to zero as the sample size (*T*) tends to infinity and the number of lags (*k*) increase such that *k/T* tends to zero.

In this way, the test is to sort the eigenvalue (λ) of the matrix $\widehat{M}(k)$, and obtain their sum as follows:

$$S_{m=m_1+m_2} = (T-k)\sum_{j=1}^{N-m} ln\left(1-\lambda_j\right) \quad (18)$$

This sum behaves asymptotically as χ^2_{N-m} distribution, in our case *N*=6 and the expected value of *m* is four.

4. Data

We use daily close Bid and Ask prices data on the exchange rate of the 20 currencies against USD and EUR (direct price[3]), from 1-july-2002 to 30-december-2011 (2,478 observations per series from Reuters-3000 XTRA 4.5 platform of inter-dealer market). As we consider Ask and Bid price, the number of series is 82. The Bid price is the last reported offer price for a money instrument at 21:50 GMT, which is part of bid/ask pair in a market maker driven market. While the Ask price is the last (and best) reported buy price from a market maker at 21:50 GMT. The difference between the Bid and Ask is known as the spread.

In Table-1 we show the sample currencies. It was selected of Triennial Central Bank Survey, Foreign exchange and derivatives market activity in 2010 issued by Bank for International Settlement (BIS, 2010, in Table 3, page 9]. We attempted at percentage importance level, and the currencies are not traded directly against EUR are excluded[4]:

[3] We use those data to avoid the problems pointed out by Pasquariello (2001) when we work on indicative quotes.

[4] In Reuters, some exchange rates against EUR are equals to triangular arbitrage prices and then, opportunities arbitrages are not possible. These currencies are: Korean won, Taiwan dollar, Brazilian real, Chilean peso and Mexican peso, because there is only a direct price of Mexican peso against EUR from 17-feb-2006.

Table 1. Sample currencies

Ranking	Code	Currency	Percentage	Accumulated
1	USD	US dollar	42.45%	42.45%
2	EUR	Euro	19.55%	62.00%
3	JPY	Japanese yen	9.50%	71.50%
4	GBP	Pound sterling	6.45%	77.95%
5	AUD	Australian dollar	3.80%	81.75%
6	CHF	Swiss franc	3.20%	84.95%
7	CAD	Canadian dollar	2.65%	87.60%
8	HKD	Hong Kong dollar	1.20%	88.80%
9	SEK	Swedish krona	1.10%	89.90%
10	NZD	New Zealand dollar	0.80%	90.70%
12	SGD	Singapore dollar	0.70%	91.40%
13	NOK	Norwegian krone	0.65%	92.05%
15	RUB	Russian rouble	0.45%	92.50%
16	PLN	Polish zloty	0.40%	92.90%
17	ZAR	South African rand	0.35%	93.25%
18	TRY	Turkish new lire	0.35%	93.60%
20	DKK	Danish krone	0.30%	93.90%
22	HUF	Hungarian forint	0.20%	94.10%
25	THB	Thailand baht	0.10%	94.20%
26	CZK	Czech koruna	0.10%	94.30%
29	IDR	Indonesian rupiah	0.10%	94.40%
30	ILS	Israeli new shekel	0.10%	**94.50%**

First, we test univariate evidence for presence of unit root in sample (logarithms of exchange rate and first difference of log-exchange rate) and then, reject stationary hypothesis for all exchange rates of data. The Table 2 shows results:

Table 2. Stationarity Tests

Currencies	ADF test of LN(exchange rate)				ADF test First difference of LN(exchange rate)			
	Ask_USD	Bid_USD	Ask_EUR	Bid_EUR	Ask_USD	Bid_USD	Ask_EUR	Bid_EUR
EUR	-2.563 [0]	-2.56 [0]	--	--	-50.09 [0]	-50.14 [0]	--	--
JPY	-0.11 [16]	-0.11 [16]	-0.68 [30]	-0.68 [30]	-13.07 [15]	-13.07 [15]	-9.45 [29]	-9.44 [29]
GBP	-1.67 [1]	-1.67 [1]	-1.27 [4]	-1.26 [4]	-47.72 [0]	-47.75 [0]	-25.18 [3]	-25.19 [3]
AUD	-1.80 [13]	-1.80 [13]	-0.23 [22]	-0.23 [22]	-13.34 [12]	-13.35 [12]	-12.69 [21]	-12.68 [21]
CHF	-1.43 [24]	-1.42 [24]	0.77 [35]	0.77 [35]	-10.51 [23]	-10.52 [23]	-10.89 [34]	-10.89 [34]
CAD	-2.11 [34]	-2.11 [34]	-1.97 [0]	-1.96 [0]	-8.28 [33]	-8.27 [33]	-49.11 [0]	-48.88 [0]
HKD	-2.57 [19]	-2.73 [18]	-2.56 [0]	-2.57 [0]	-11.01 [18]	-10.95 [18]	-50.23 [0]	-50.27 [0]
SEK	-2.64 [9]	-2.71 [8]	-1.66 [49]	-1.64 [49]	-16.48 [8]	-16.95 [7]	-6.89 [48]	-6.99 [48]
NZD	-2.40 [4]	-2.41 [4]	-1.87 [3]	-1.87 [3]	-24.81 [3]	-24.78 [3]	-29.81[2]	-29.84 [2]
SGD	-0.59 [21]	-0.59 [21]	-1.31 [3]	-1.31 [3]	-12.08 [20]	-12.15 [20]	-31.07 [2]	-31.09 [2]
NOK	-2.42 [0]	-2.42 [0]	-2.55 [18]	-2.55 [18]	-36.68 [1]	-36.75 [1]	-12.08 [17]	-12.09 [17]
RUB	-2.05 [19]	-2.05 [19]	-1.75 [0]	-1.74 [1]	-9.06 [18]	-9.06 [18]	-51.46 [0]	-51.54 [0]
PLN	-2.13 [26]	-2.04 [15]	-1.45 [6]	-1.45 [6]	-9.61 [25]	-9.69 [25]	-23.89 [5]	-23.90 [5]
ZAR	-2.53 [15]	-2.63 [11]	-1.66 [0]	-1.64 [0]	-13.47 [14]	-15.26 [10]	-50.54 [0]	-30.04 [2]
TRY	-1.71 [4]	-1.68 [4]	-1.38 [1]	-1.32 [3]	-24.68 [3]	-24.61 [3]	-53.36 [0]	-30.29 [2]
DKK	-2.58 [0]	-2.58 [0]	-2.81 [5]	-2.80 [5]	-50.34 [0]	-50.39 [0]	-20.46 [5]	-21.68 [4]
HUF	-2.54 [0]	-2.51 [0]	-1.20 [6]	-1.23 [6]	-49.95 [0]	-49.69 [0]	-23.28 [5]	-23.17 [5]
THB	-1.12 [49]	-1.16 [49]	-2.26 [17]	-2.08 [0]	-7.40 [48]	-7.16 [48]	-10.63 [16]	-51.18 [0]
CZK	-1.72 [9]	-1.70 [0]	-1.15 [33]	-1.13 [33]	-16.35 [8]	-11.17 [15]	-8.82 [32]	-8.78 [32]
IDR	-2.26 [31]	-2.24 [31]	-2.37 [1]	-2.39 [1]	-9.57 [30]	-9.59 [30]	-57.97 [0]	-59.43 [0]
ILS	-1.61 [1]	-1.49 [7]	-2.72 [3]	-2.72 [3]	-47.01 [0]	-19.78 [6]	-31.58 [2]	-31.67 [2]

Note: ADF tests are estimated with constant and a [number of lags] by information criteria AIC. Critical values of rejection non-stationary are -2.86 and -3.44, at confidence levels of 5% and 1%, respectively.

We observe that the logarithm of exchange rates are non-stationary, while the first difference of log (returns) is stationary, therefore, all data series in logarithms have a unit root. Besides, it should be noted that the currencies of the Euro-zone have higher lags against the euro to the dollar, while for the rest happens on the contrary, except for NZD. In addition, except for the exchanges rates THB/EUR and ZAR/USD, the bid prices showed more lags than ask. The exchange rates showing highest difference in the number of lags between the bid and ask prices are: IDR/USD and RUB/EUR.

In short, data show heteroskedasticity and autocorrelation, which fully justifies the analysis of cointegration relations not only to lags that indicate the information test (AIC), but also from significant lags (F-test), incorporating multivariate heteroskedasticity (bootstrap BEKK) and by a supplementary dynamic factorial analysis (PP). The results of the different analysis of cointegration can be seen in Table-3:

Table 3. Cointegration Rank with Ask and Bid prices

	Analysis for lag of AIC				Analysis for lag of F-test						Analysis bootstrap BEKK		Analysis of common factors test		
Currency	AIC lags	rank	test trace	pvalue	last lag	F-test last lag	F-prob	rank	test trace	pvalue	rank	pvalue bootstrap	common factors	PP test	pvalue
JPY	11	4	10.55	[0.593]	42	1.702	[0.006]**	2	50.41	[0.102]	2	[0.117]	4	8.459	[0.489]
GBP	11	4	10.13	[0.633]	46	1.578	[0.015]*	2	52.776	[0.093]	2	[0.143]	4	23.64	[0.491]
AUD	26	3	29.75	[0.173]	46	1.538	[0.021]*	2	53.344	[0.064]	2	[0.101]	4	3.564	[0.468]
CHF	11	4	9.30	[0.711]	32	1.82	[0.002]**	2	50.473	[0.101]	2	[0.138]	4	4.612	[0.329]
CAD	7	4	13.40	[0.340]	42	1.19	[0.012]*	2	53.159	[0.071]	2	[0.124]	4	2.827	[0.587]
HKD	11	4	14.02	[0.295]	47	1.755	[0.004]**	2	52.822	[0.081]	2	[0.104]	4	7.702	[0.103]
SEK	6	4	11.27	[0.525]	50	1.849	[0.002]**	2	45.515	[0.234]	2	[0.229]	4	5.686	[0.224]
NZD	8	4	11.71	[0.483]	43	1.736	[0.004]**	2	47.393	[0.174]	2	[0.236]	4	1.163	[0.884]
SGD	11	4	13.48	[0.334]	53	1.83	[0.002]**	2	51.025	[0.090]	2	[0.176]	4	1.385	[0.239]
NOK	11	4	14.46	[0.265]	46	2.381	[0.000]**	2	49.771	[0.115]	2	[0.277]	4	5.852	[0.210]
RUB	12	4	9.33	[0.708]	49	1.983	[0.000]**	2	49.805	[0.114]	2	[0.165]	4	7.919	[0.095]
PLN	12	4	12.39	[0.423]	44	1.638	[0.010]**	2	52.168	[0.072]	2	[0.086]	4	7.689	[0.104]
ZAR	8	4	16.17	[0.169]	50	1.698	[0.009]**	2	51.94	[0.092]	2	[0.154]	4	7.214	[0.112]
TRY	11	4	10.98	[0.552]	37	1.774	[0.003]**	2	47.721	[0.165]	2	[0.093]	4	8.288	[0.089]
DKK	11	4	14.99	[0.232]	33	1.487	[0.030]*	2	48.93	[0.133]	2	[0.241]	4	4.491	[0.344]
HUF	12	4	11.24	[0.527]	43	1.756	[0.003]**	2	42.135	[0.373]	2	[0.462]	4	1.752	[0.781]
THB	18	2	53.09	[0.060]	39	1.637	[0.010]**	2	39.675	[0.492]	2	[0.596]	4	6.857	[0.144]
CZK	8	4	19.32	[0.066]	57	1.651	[0.008]**	2	38.119	[0.522]	2	[0.505]	4	3.087	[0.5433]
IDR	16	4	15.65	[0.195]	55	1.912	[0.001]**	2	40.777	[0.501]	2	[0.654]	4	5.279	[0.260]
ILS	11	4	16.93	[0.137]	40	1.588	[0.014]*	2	40.283	[0.461]	2	[0.439]	4	7.729	[0.102]

Note: Columns 2-5 shows the results (using AIC criteria to select lags) of traditional cointegration analysis. Columns 6-8 show an F-test to find the last significant delay [(*) <5% or (**) <1%] in the VAR. Columns 9-11 show the results by the trace test using significant lag in F-test. Columns 12 to 13 show the results of the cointegration analysis by bootstrapping technique with heteroskedasticity. Columns 14-16 show the result of dynamic factorial test, that is, the number of variables (six exchange rates) minus the number of common dynamic factors indicates the cointegration rank.

As shown, for each six set of exchange rates (bid-ask price of each currency against USD and EUR more EUR/USD bid-ask) only using the number of delays that AIC indicates the number of cointegration relationships are 4, while for bootstrap BEKK there are 2 cointegration relations, and for PP test 4 common factors (or equivalently, 2 cointegration relations). Therefore, to test whether there are triangular arbitrage opportunities, we consider that there are two cointegration relations, one for the bid price and the other to ask.

5. Results

The null hypothesis of the lack of arbitrage opportunities is: $(\beta_{0,1} - \beta_{0,2}) = 0$. In Table-4, we show LR-test on this hypothesis:

Table 4. LR test on constraints of arbitrage opportunities

Currency	Beta constraint *Chi^2(8)*	Beta+cost constraint *Chi^2(9)*	Beta vs. Beta+cost *Chi^2(1)*
JPY	3.28 [0.916]	6.1 [0.73]	2.82 [0.093]
GBP	5.76 [0.674]	9.2 [0.419]	3.43 [0.064]
AUD	6.66 [0.574]	8.6 [0.475]	1.94 [0.164]
CHF	4.47 [0.812]	6.86 [0.651]	2.39 [0.122]
CAD	10.94 [0.205]	13.21 [0.153]	2.27 [0.132]
HKD	1.26 [0.996]	4.32 [0.889]	3.06 [0.081]
SEK	8.83 [0.357]	11.93 [0.218]	3.1 [0.078]
NZD	10.46 [0.234]	11.51 [0.242]	1.05 [0.306]
SGD	9.17 [0.328]	11.42 [0.248]	2.24 [0.134]
NOK	9.91 [0.272]	12.07 [0.21]	2.16 [0.142]
RUB	11.04 [0.199]	13.73 [0.132]	2.69 [0.101]
PLN	7.97 [0.437]	10.81 [0.289]	2.85 [0.092]
ZAR	4.04 [0.853]	5.6 [0.78]	1.55 [0.213]
TRY	12.85 [0.117]	15.89 [0.069]	3.04 [0.081]
DKK	8.24 [0.41]	11.5 [0.243]	3.26 [0.071]
HUF	7.57 [0.477]	11.27 [0.258]	3.7 [0.059]
THB	11.73 [0.164]	14.97 [0.092]	3.24 [0.072]
CZK	11.05 [0.199]	13.96 [0.124]	2.91 [0.088]
IDR	11.82 [0.16]	15.4 [0.081]	3.58 [0.058]
ILS	8.51 [0.385]	11.76 [0.227]	3.25 [0.072]

Note: This test is defined as: $LR = -2 \cdot [loglik_r - loglik_N] \sim \chi^2_{df}$, where *loglik* is log-likelihood for general model (*N*) and model with *r* constraints; *df* is degree freedom. First column shows LR-test between expression (9) and this general expression modified with beta constraint (10). Second column is LR-test between expression (9) and modified model with beta and constant (cost) constraints. Third column shows LR-test between modified expression with beta constraint, but with different and equal constants (cost). The hypothesis null is rejected at 5% (*) or at 1% (**).

The results show that both price cointegration relationships (ask-bid) are defined and that the assumption of arbitrage opportunities is rejected for all currencies.

Table-5 shows the daily rate of transaction costs estimated for each case:

Table 5. Daily rate of transaction cost

Currency	without restrictions				beta restrictions				cost equal	
	BID	t-value	ASK	t-value	BID	tvalue	ASK	t-value	CONSTANT	t-value
JPY	0.02%	0.87	0.02%	0.92	0.02%	10.43**	0.02%	10.46**	0.02%	9.53**
GBP	0.02%	0.53	0.02%	0.26	0.02%	23.52**	0.02%	36.43**	0.02%	4.68**
AUD	0.04%	1.52	0.04%	2.57**	0.04%	28.94**	0.04%	41.34**	0.04%	9.28**
CHF	0.03%	6.15**	0.03%	6.16**	0.03%	2.37*	0.03%	2.38*	0.03%	2.82**
CAD	0.05%	13.84**	0.06%	13.76**	0.04%	0.03	0.04%	0.03	0.04%	5.40**
HKD	0.01%	11.43**	0.01%	11.49**	0.01%	11.07**	0.02%	11.07**	0.01%	9.58**
SEK	0.07%	16.40**	0.07%	16.22**	0.07%	5.52**	0.07%	5.55**	0.07%	5.57**
NZD	0.03%	3.92**	0.03%	4.13**	0.03%	41.01**	0.03%	38.02**	0.03%	6.31**
SGD	0.05%	17.13**	0.05%	17.14**	0.05%	11.84**	0.05%	11.86**	0.05%	5.16**
NOK	0.05%	2.67**	0.05%	2.67**	0.04%	3.12*	0.05%	3.14**	0.04%	3.08**
RUB	0.03%	4.30**	0.04%	4.24**	0.03%	15.45**	0.03%	15.42**	0.03%	14.02**
PLN	0.10%	0.17	0.10%	0.15	0.10%	7.04**	0.10%	11.92**	0.10%	10.97**
ZAR	0.16%	3.34**	0.16%	4.46**	0.17%	4.08**	0.17%	4.72**	0.17%	2.43*
TRY	0.16%	20.91**	0.16%	20.95**	0.15%	2.00*	0.15%	2.00*	0.15%	8.26**
DKK	0.02%	10.66**	0.02%	10.82**	0.02%	4.64**	0.02%	4.67**	0.02%	2.88**
HUF	0.11%	24.14**	0.11%	24.04**	0.12%	13.97**	0.12%	14.23**	0.12%	7.33**
THB	0.14%	0.93	0.14%	3.47**	0.13%	14.80**	0.13%	15.46**	0.13%	7.77**
CZK	0.10%	0.59	0.10%	0.6	0.10%	25.97**	0.01%	39.85**	0.11%	6.95**
IDR	0.14%	12.50**	0.14%	13.31**	0.13%	13.79**	0.14%	13.17**	0.13%	8.13**
ILS	0.14%	25.59**	0.14%	25.49**	0.14%	2.81**	0.14%	2.82**	0.14%	15.00**

Note: First column shows the rates of transaction cost to expression (9). The rate in second column are estimated with expression (9) and the constraints of expression (10). Third column shows the estimated rates under expression (9), (10) and an additional restriction on equality of transaction cost of ask and bid prices, i.e. $\beta_{0,1} = \beta_{0,2}$.The null hypothesis on transaction cost is rejected at 5% (*) or at 1% (**).

We observe that the model with a cost equal to the ask and bid ratio is significant in all cases, and the cost is higher when the currency has less weight in the global market.

6. Conclusions

This paper studies the foreign exchange market efficiency testing for the long-run triangular arbitrage relationship among the spot exchange rate. The results show evidence of no arbitrage opportunities. For doing this test, in opposite of others authors, on the one hand, we have considered the micro-structure of spot exchange market (daily bid and ask prices), i.e. the implicit transaction costs; and on the other hand, using a unique database that represents more at 94% of total currency market (USD, EUR and others 20 currencies).

To overcome the problems arising from the use of high frequency data in traditional cointegration test, we have employed a cointegrated VAR with a covariance matrix heteroskedastic (BEKK) and we test the hypothesis according to a bootstrap methodology and validate the results by a test of dynamic factors.

Our results lend empirical support to authors with respect to cointegration relationship among exchange rates. But, contrary of others, we conclude that the null hypothesis of non-arbitrage opportunities cannot be rejected. In conclusion, the spot currency markets are cointegrated and so long-run arbitrage free, and as consequence that, efficient.

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